Vortical Wakes Over a Prolate Spheroid

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The vortex-lattice method is employed to calculate the inviscid flow and the development of separated vortex sheets over a prolate spheroid. An approximate boundary-layer method based on the assumption of local similarity is used to calculate the line of open separation. A condition of vortex shedding along the separation line is proposed. The two methods of calculation, viscous and inviscid, interact through the line of separation that is allowed to displace as the wake grows. Results are compared with flow visualization data for laminar separation and pressure data for turbulent separation.

I. Introduction

AERODYNAMIC problems involving massive regions of separated flow in three dimensions can be treated today numerically by methods of discrete vortex dynamics. Navier-Stokes codes, instead, require much longer computer time and space to generate the solution to a practical problem, and in some cases fail to predict separated flows accurately beyond Reynolds numbers of a few thousands, even for two-dimensional flows. In two dimensions, a variety of problems have been studied with the method of discrete vortex dynamics, as for example the stability of free shear layers and the wake structures of thin plates or cornered bodies. More closely related to the present contribution are works considering the problems of separation over bluff bodies such as the circular cylinder.[1-2] Two-dimensional methods have been refined to include redistribution of vorticity, models for viscous dissipation, etc. (see review and critique in Ref. 2). The first efforts to consider separated regions over elongated three-dimensional bodies were based on the crossflow-plane analogy, which employs two-dimensional solutions in cross sections of the body. In three-dimensional problems, a class of vortex methods is known as "vortex-lattice methods." Problems involving sharp-edge separations have been treated in this way with considerable success for steady[3-4] and unsteady[5-6] flows.

Three-dimensional flows involving separation over smooth surfaces have been considered only very recently. Fiddes[7] expanded on a viscous-inviscid interaction method to obtain solutions for separated flow over a cone. This procedure, however, is limited to conical flows over slender bodies. Thrasher[8] studied the flow over a semi-infinite body with a tangent-ogive nose and a cylindrical afterbody. He employed an iterative version of a vortex-lattice scheme and assumed that the separation line coincided with a generator of the body. Almosnino and Rom[9] considered the flow about a very similar semi-infinite body, and employed again an iterative method to achieve convergence of the wake vortices. However, they modeled the separated vortex sheets by only four vortex lines on each side emanating from very narrow segments, aligned again with generators of the body. More recently, a refined version of this method, which relaxed a few of the constraining assumptions but still required the specification of the line of separation, appeared in Ref. 10.

In all investigations of three-dimensional bluff-body separated flows, the body was assumed to be semi-infinite. Bodies closed fore and aft are known to give rise to singular matrices in vortex-lattice solutions. Moreover, with the exception of the degenerate case considered by Fiddes,[7] all other investigators assume that the position of separation is known and that, in fact, it coincides with a meridional line of axisymmetric bodies. In the present paper we report on our efforts to relax the above restrictions. This was achieved by coupling the vortex-lattice solution with an approximate method of calculating the three-dimensional boundary layers. Moreover, we considered the development of the wake in a quasisteady domain and allowed the viscous and inviscid fields to interact. The line of separation over a finite body was readjusted at each time step and was found to tend to the position predicted experimentally for a fully developed flow.

Three-dimensional laminar boundary layers have been calculated by a variety of methods in the past decade. Work in this area was essentially initiated by Wang.[11-13] This was followed by contributions of Cebeci et al.,[14] Patel and Baek,[15] Tai,[16] Ragab,[17] and Radwan and Lekoudis.[18] The present authors have developed an approximate method that is described in Ref. 19, and is used here to compute the separation line. Although this method is not as accurate, it is very efficient, requiring very little computer time and is well suited for coupling with a panel method. The method is based on a solution proposed by Sears,[20] who made one of the pioneering contributions in the area and to whom this paper is dedicated.

This method was employed essentially in order to generate the line of separation. Although very crude, it proved to compare favorably with other more involved methods. Extensive comparisons are included in Ref. 19. As examples, we present here comparisons of the location of the line of separation with the results of Stock[21] and Geissler[22] in Fig. 1, and the results of Wang[23] and Cebeci et al.[14] in Fig. 2. In these figures, x/a is the reduced axial distance and ϕ is the meridional angle, measured from the windward plane.

II. Inviscid Flow Calculations

The outer flow and the separated vortex sheet are predicted by the vortex-lattice method. This is based on the Biot-Savart law that for the case of a vortex segment of constant strength γ and finite length induces a velocity given by (Fig. 3)

$$\bar{V}(\bar{r}) = e\,\frac{\gamma}{4\pi h}\,(\cos\theta_1 - \cos\theta_2) \tag{1}$$

where e is a unit vector normal to the plane defined by the point of interest and the vortex segment.

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Appropriate distributions of vorticity along fixed or moving grids can be used to generate solutions of incompressible flows in two or three dimensions, with or without separation. As a test for three-dimensional bluff bodies, we considered three basic problems that accept exact closed-form analytical solutions, readily available for comparison: the flow about 1) a sphere, 2) a prolate spheroid, and 3) a general ellipsoid. To solve these problems, vortex lattices have been defined with their nodes on the surface of the body. The strength of each vorticity segment, usually referred to as branch circulation, is denoted by the symbol γ. Most investigators working in this area[3,5,6] consider the basic unit of vorticity as a constant-strength "ring" vortex lying along the edges of a quadrilateral panel of the vortex lattice. The strength of this vortex for a given panel is denoted the loop circulation G. The branch circulation γ is simply the difference of the loop circulations of its two neighboring panels G_1 and G_2

$$\gamma = G_1 - G_2 \tag{2}$$

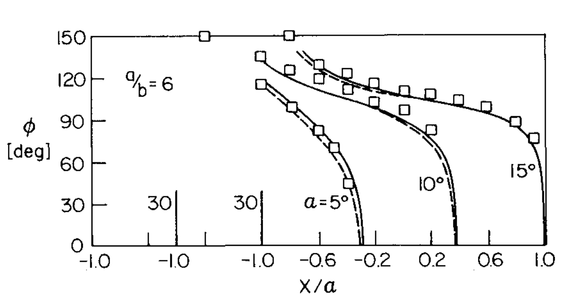

Fig. 1 Lines of separation for a prolate spheroid with axes ratios $a/b = 6$ and angles of attack $\alpha = 5$, 10, and 15 deg: ———, Stock;[21] – – – –, Geissler;[21] □, present method.

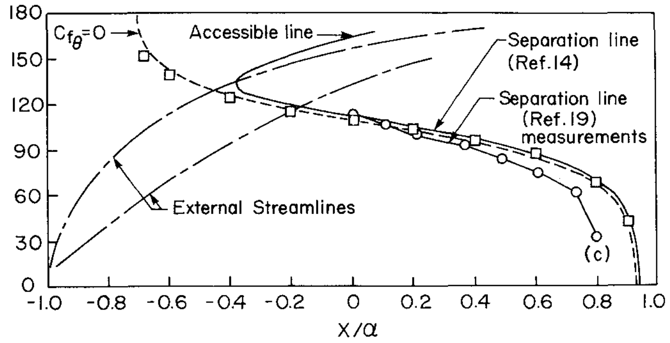

Fig. 2 Lines of separation and external streamlines for a prolate spheroid of $a/b = 6$ at an angle of attack $\alpha = 15$ deg as reported in Ref. 14: □, present approximate method; -o-o-o-o, experimental data from Ref. 19.

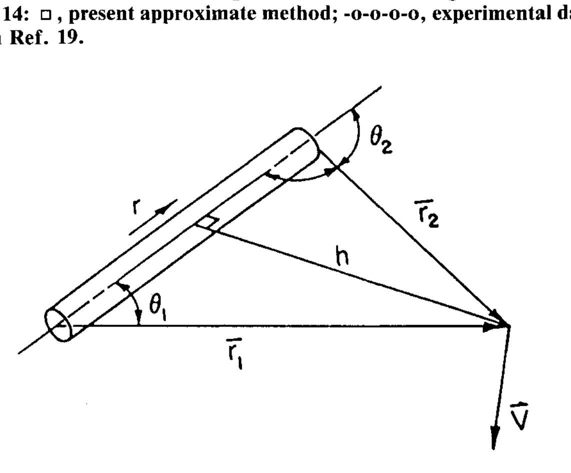

Fig. 3 Velocity induced at a point due to a vortex segment.

The branch circulations thus defined generate automatically a divergence-free bound vortex sheet.

Consider now a case of N panels with N unknown loop circulations G_j, $j = 1,\ldots,N$. These quantities can be specified by imposing the no-penetration condition at N points on the body. We chose as control points, the centroids of the panels, but we project these points on the actual surface of the body under consideration. The influence matrix A_{ij} is first constructed. An element of this matrix is the value of the normal velocity at the ith control point due to vorticity distributed around the jth panel with a unit loop circulation. The no-penetration condition then is simply expressed as

$$\sum_{i=1}^{N} A_{ij}\, G_j = \bar{V}_\infty \cdot \bar{n}_i \tag{3}$$

where $\bar{V}_\infty$ is the freestream velocity and n_i is the normal to the body at the point i. The solution of this system for the quantities G_j can generate the flowfield about the body under consideration.

We eliminate the indeterminacy associated with closed surfaces by relaxing the no-penetration condition near the tail of the body. The error thus introduced is negligible. Results for attached three-dimensional flow obtained by this method compare favorably with exact solutions for different configurations such as the sphere, the prolate spheroid, and the ellipsoid.[19] Separated flows can be calculated by approximating the separated vortex sheets again by vortex lattices.[13-16] This is discussed in detail in Sec. III.

In the present method we consider impulsively started flows. The steady state is then obtained as a limit for large times. The intent here is to generate a steady-state solution rather than capture the true unsteady development of an impulsively started motion. At present, due mainly to limitations in computing time, we have not obtained steady-state solutions. However, the present results display some very encouraging characteristics.

The Kelvin-Helmholtz theorem dictates that vorticity be transported with the local velocity. This condition automatically generates a force-free wake and is used in a manner similar to the one described in earlier references.[5-6] The main difference here is that the wake is initiated along a line of separation that is determined by a viscous calculation and is free to readjust at each time step.

The shed vorticity in the form of two rows of panels at the root of the separated vortex sheets are determined at each time step from viscous-inviscid interaction, as described in Sec. IV. In the next time step, this row of panels is convected away from the body and a new row of panels is generated in the same way. The free vortex panels must be accounted for in the calculation of the velocity field. Equation (3) must be modified to include $\bar{V}_w$, the velocity induced by the wake panels

$$\sum_{i=1}^{N} A_{ij}\, G_j = (\bar{V}_\infty + \bar{V}_w) \cdot \bar{n}_i \tag{4}$$

It should be noted that the matrix A_{ij} is invariant for a certain body configuration, but the calculation of the velocity $\bar{V}_w$, although tedious and time-consuming, must be repeated at each time step. Finally, the system of Eq. (4) is nonsingular if framed as discussed earlier, and can be solved with the same routine used to solve Eq. (3).

III. Viscous-Inviscid Interaction and Results

In the present problem we allow the boundary layer to interact with the outer flow only through separation. The strength of the vorticity shed at separation and the location of the line of separation control the development of the free vortex sheets that roll up forming the wake. The structure of the flow at separation and the mathematical modeling that allows the two solutions, the viscous and the inviscid, to interact with each other are of crucial importance.

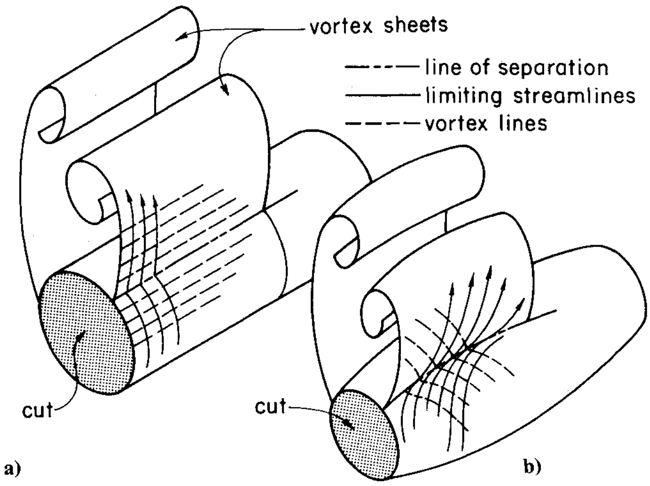

Fig. 4 Schematic of the flow near separation: – – – –, wall vortex lines; ———, skin-friction lines; – · · · –, separation line; a) two-dimensional flow, and b) three-dimensional flow.

The topography of skin-friction lines has been studied extensively in three-dimensional boundary-layer flows.[23,24] Experimental evidence and numerical information indicate that the skin-friction lines merge along the line of separation. Recent careful boundary-layer calculations[14] point to the direction that the separation line is an envelope of skin-friction lines and, therefore, within the framework of uninteracted boundary-layer theory, a singular behavior should be expected there.

This is a subtle point and requires more clarification. Uninteracted boundary-layer solutions are known to behave singularly at separation in two- or three-dimensional flows (Williams[25]). Nature, of course, does not accept singularities and, in fact, it has been shown that the correct mathematical model, i.e., the full Navier-Stokes equations, are nonsingular at separation. Yet, boundary-layer solutions predict with reasonable accuracy the location of separation, if not the actual fluid behavior there. If interacted properly with the outer flow, the singularity can actually be removed, but this is not the issue here. Our aim is to demonstrate that an iterative process whereby the developing outer flow is employed to solve the boundary-layer equations and calculate the line of separation leads to calculated locations of the separation line that are close to its experimentally determined position.

It is well-known that the vortex lines on the surface of the body are orthogonal to the skin-friction lines. This implies that the surface vortex lines meet the line of separation at an angle of 90 deg and lift off the body surface also perpendicular to the line of separation, since the skin-friction lines become tangent to the line of separation. Incidentally, this is contrary to the common concept of two-dimensional separation where, in principle, surface vortex lines are parallel to the line of separation. This idea is schematically depicted in Fig. 4.

The picture described above and shown in Fig. 4 is actually the limit of the behavior of streamlines and vortex lines, as the distance from the wall tends to zero. For the purely two-dimensional case, the line of separation is a line of zero vorticity. But the total amount of vorticity contained in the boundary layer, even in the purely two-dimensional case, is not zero at separation. In fact, it is the amount that the boundary layer sheds in the wake.

For two-dimensional flow, the rate at which vorticity is convected in the boundary layer and is eventually shed at the point of separation was pointed out by Fage and Johansen[26] to be

$$\frac{d\Gamma}{dt} = \int_0^\delta \Omega_y u \, dz \tag{5}$$

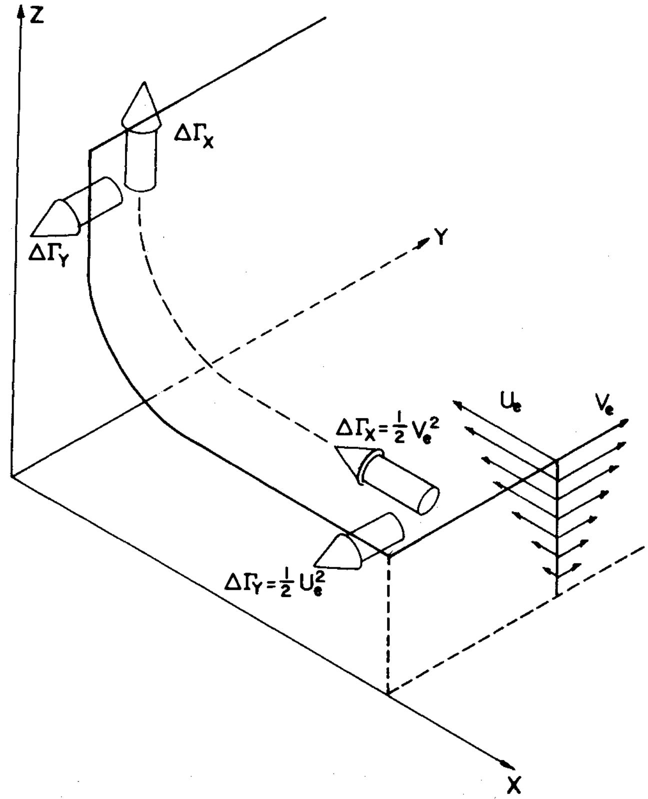

Fig. 5 The vorticity components before and after separation; here the y axis is aligned with the line of separation.

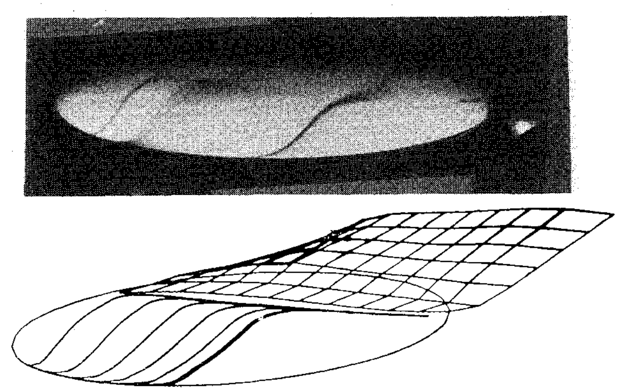

Fig. 6 Comparisons of visualized and calculated skin-friction lines for a prolate spheroid of axes ratio 1:4.

This is essentially the vorticity flux across the boundary layer. Within the boundary-layer approximation and for a finite time step this becomes

$$\Delta\Gamma = \tfrac{1}{2} V^2 \Delta t \tag{6}$$

with V the edge velocity. If V is the edge velocity at separation, then Eq. (6) defines the strength of a nascent vortex released at intervals Δt. Saffman and Schatzman[27] discuss the issue in detail and reference a large number of experimentalists who have estimated indirectly the fraction of $V^2/2$, which is actually shed in the free shear layer. They also reference many numerical contributions that employed this idea to solve problems involving two-dimensional separating flows.

Here, we propose the extension of this idea to three-dimensional boundary layers. Similar integrations across the boundary layer yield the strength of nascent vortices in the x and y

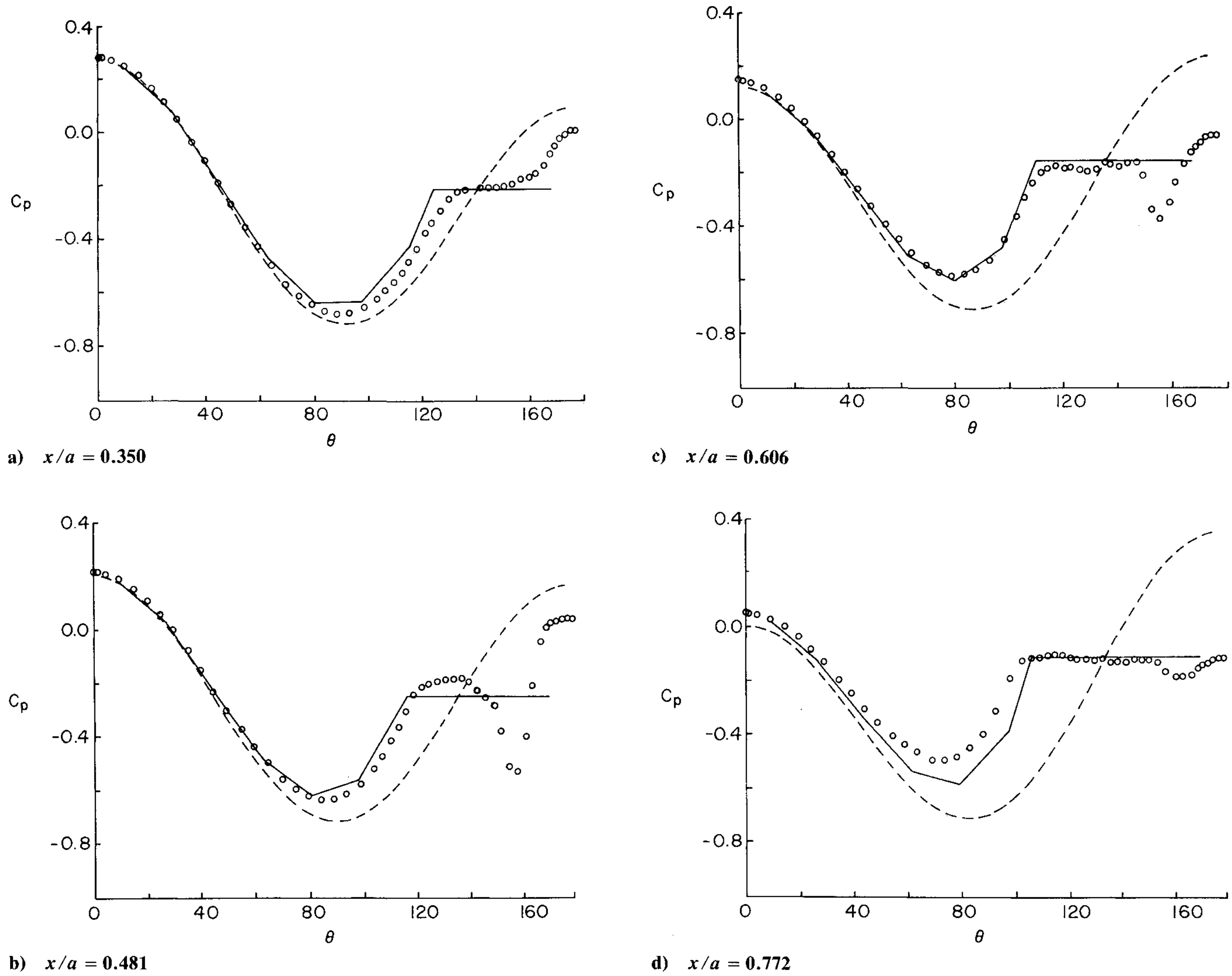

Fig. 7 Pressure distributions on a prolate spheriod of axes ratio 1:6: ———, present calculations; – – – –, potential solutions; 0, measurements of Ref. 28.

direction

$$\Delta\Gamma_x = \tfrac{1}{2} U_e^2 \Delta t, \qquad d\Gamma_y = \tfrac{1}{2} V_e^2 \Delta t \tag{7}$$

where x and y are normal and parallel to the line of separation, and U_e, V_e are the edge velocities along these directions, respectively.

What is of great importance and interest here is that vorticity in the boundary layer is parallel to the wall, but not necessarily parallel to the line of separation. This is because maximum vorticity flux is in a direction normal to the edge velocity which, however, is not normal to the line of separation. If we need to discretize the nascent vorticity, then we can simply resolve it in components parallel and perpendicular to the line of separation, but always parallel to the surface of the body.

Here we model the free shear layer by a free vortex lattice. The component $\Delta\Gamma_y$ is then released in the wake with its orientation preserved. The component $\Delta\Gamma_x$ is preserved in magnitude, but is turned appropriately to realign with the direction of the free vortex sheet, as shown schematically in Fig. 5. The justification for such an assumption is the fact that except for diffusion and dissipation, vorticity vectors drift with the flow as if they were made of fluid particles. Diffusion and dissipation are negligible for small distances. In our numerical scheme the components $\Delta\Gamma_y$ and $\Delta\Gamma_x$ parallel and perpendicular to the line of separation, respectively, were discretized into a ribbon of rectangular elements. The surface of the ribbon was assumed to be parallel to the surface of the body at an arbitrary distance of $0.2R$ where R is the radius of the cross section of the body at the station under consideration. More properly, this distance should be proportional to the displacement thickness of the boundary layer. The outer nodes of the ribbon were allowed to be convected, and a new ribbon of vorticity was generated according to the above scheme. In this way a free vortex sheet was generated and allowed to grow.

The most upstream position of the line of separation was assumed arbitrarily. However, it was found that vorticity led into the vortex sheet at this point is very small. As a result, the global effect of this assumption is negligible.

The solution was constructed as follows. A set of skin-friction lines based on the exact potential flow solution was calculated first by the approximate boundary-layer method. The envelope of the skin-friction lines defined the line of separation. The strength of the nascent vortices was then calculated and allowed to drift with the local velocity. The inviscid flow problem, namely Eq. (4), were solved again to determine the new loop circulations on the body. With the updated inviscid flow, a new set of skin-friction lines was calculated to determine a new envelope, namely the new position of the line of separation. Once again, the strength of the nascent vortices was calculated. The free vortices were then convected by increments proportional to the local velocities. The process was then repeated for a total of six time steps. It is clearly desirable to allow the calculation to march to higher values of time, but this was not possible because of computer time limitations.

In Fig. 6 we display the direct comparison of flow visualizations with our calculation. The streaklines in this figure were obtained by dyes released in the windward side of the model.[19] The dye streaks aligned more or less with the skin-friction lines. Moreover, they turn sharply along the line of separation and lift off along the vortex sheet.

No pressure data is available for low Reynolds number flows. To test how accurately the vortex lattice method would predict pressure distributions, we calculated the flow over a prolate spheroid of axes ratio 1:6 at an angle of attack $\alpha = 15$ deg, in order to match the data of Ref. 28. In these calculations, the experimental line of separation is input and only the vortex lattice portion of the code is used.

Results are shown in Fig. 7. It was found that simulating the separated vortex sheets with only four to six ribbons generates a pressure imprint on the solid surface with large circumferential variations, a clear indication of potential vortices. This behavior was arbitrarily smoothed by assuming that, beyond separation, the pressure remains uniform and equal to the value of pressure at separation. It is observed in Fig. 7 that the pressure variation on the attached region of the flow is quite accurately predicted by the method.

IV. Conclusions

The present calculations have further demonstrated the potential of the vortex-lattice method. For the first time, an approximate boundary-layer solution was coupled with the vortex-lattice solution to determine the location of separation and its displacement as the wake grows and rolls up. It was demonstrated that the strength of the shed vortex can be determined by simple approximate formulas that essentially represent integrals of boundary-layer profiles.

The pressure distribution on the attached region of the boundary layer can be predicted accurately with a very crude simulation of the separated vortex sheets. However, the pressure underneath the rolling separated sheets displays large spikes, which are clear indication of potential vortices and are not present in the actual flow. Apparently, in the latter case, a more uniform distribution of the vorticity and perhaps the presence of smaller eddies smooth out the pressure into distributions that are almost uniform. Modeling this situation with discrete vorticity will probably require a very dense grid. This in turn may involve stability problems, which will have to be handled by some type of redistribution of vorticity on the vortex sheet.

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